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TWO RECOMMENDATIONS FOR THE ACQUISITION AND GROWTH OF RELIABLE SYSTEMS¹

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This paper presents two recommendations for improving the acquisition and growth of reliable systems that support the intent of DoDI 5000.02 and ANSI/GEIA-STD-0009.

During the proposal evaluation and selection process, use a metric based on a Historical Observed Reliability Ratio (HOR-R, pronounced “horror”) of the potential supplier’s predicted or assessed reliability measure to its observed field reliability value.

- › Consistent HOR-R values of less than or equal to 1.0 provide confidence that the supplier has a repeatable process for translating its prediction/assessment methodology of choice into correlated field experience that meets or is better than the reliability requirement, representing limited reliability and life cycle cost risk to the customer.
- › HOR-R values greater than 1.0 indicate potential risk to the customer, in that the supplier has not demonstrated an ability to achieve reliability requirements in the field based on its prediction/assessment techniques, implying increased

reliability and life cycle cost risk.

- › Inability of a supplier to provide any HOR-R value based on past performance represents an unknown level of reliability and life cycle cost risk to the customer.
- › Any reliability prediction or assessment technique can be used, e.g., empirical handbooks, physics-of-failure (PoF), etc., since the effectiveness of the metric is not based on the ability of the approach to generate a “suitable” number.
- › The metric can be applied to requirements based on Mean Time Between Failure (MTBF), Mean Time to Failure (MTTF), Reliability (R(t)), Operational Availability (A_o), etc.

Extend the definition of reliability growth A-Mode and B-Mode failures [1, 2] to include classifications of “Unanticipated Failure Mode” and “Unexpected Failure Mode”.

- › The larger the percent contribution of Unanticipated Failure Modes to Total Failure Modes, the less robust the Design for Reliability (DFR) process is in proactively identifying

¹ This article is adapted, with permission, from the 2010 Proceedings of the Annual Reliability and Maintainability Symposium. © 2010 IEEE.

failure modes prior to testing. Corrective action is based on an evaluation of current DFR analyses, modeling and simulation processes to improve their ability to identify failure modes.

- › The larger the percent contribution of Unexpected Failure Modes to Total Failure Modes, the less effective the DFR process is in mitigating known failure modes. Corrective action is to improve reliability design practices, rules, procedures, etc., to more effectively mitigate identified failure modes prior to test.

These two recommendations, and the corrective actions they initiate, provide benchmarks to improve both the effectiveness of acquisitions in reliability and life cycle cost risk avoidance, and the ability of DFR activities to proactively identify and mitigate failure modes prior to their more costly discovery during testing or field use.

Introduction

The US Department of Defense Instruction (DoDI) 5000.02, "Operation of the Defense Acquisition System", dated 02 December 2008, and the Information Technology Association of America (ITAA) ANSI/GEIA-STD-0009-2008, "Reliability Program Standard for Systems Design, Development, and Manufacturing", dated 13 November 2008, were developed and released to reflect the revitalized, aggressive posture of the DoD in acquiring and growing reliable systems.

Specifically, DoDI 5000.02, Enclosure 2, Paragraph 5.d.5 states that "(Program Managers)...shall formulate a viable Reliability, Availability, and Maintainability (RAM) strategy that includes a reliability growth program as an integral part of design and development." Additional reliability program requirements during the Manufacturing, Production, Deployment and Operations and Support phases are also defined within Enclosure 2.

The ANSI/GEIA Standard was developed as a joint government/industry replacement for the cancelled MIL-STD-785B, "Reliability Program for Systems and Equipment Development and Production", with the intent to align reliability management, design and testing best practices with reliability methods that provide the most value and the least risk in terms of achieving reliable products."

The recommendations made in this paper provide quantitative metrics for achieving these stated objectives.

Recommendation: A metric for acquiring reliable systems

The first metric deals with decisions made by a customer when selecting between competing suppliers during an acquisition based on a perceived ability to meet the reliability requirements. A preliminary estimate of this ability may be required based on a customer-specified prediction or assessment methodology, such as MIL-HDBK-217. All bidders then apply this method and the customer picks the winner based on the "best value" reliability (relative to functional perform-

ance, cost and schedule requirements). This approach may not result in compliance with the reliability requirements, and can result in significant cost risk to a program over its total life cycle.

As a hypothetical example, suppose that a Request for Proposal

Supplier	MIL-HDBK-217 Predicted MTBF (in Hours)	Cost Proposal Relative Rank (1 = lowest cost)
A	1845	2
B	2000	1
C	2304	4
D	2840	5
E	3080	3

Table 1 – Reliability Prediction Responses to Hypothetical 1500-Hour MTBF Requirement

(RFP) includes a 1500-hour MTBF requirement using MIL-HDBK-217F, Notice 2. Five potential suppliers respond, as shown in Table 1. Assume that all other technical performance and program schedule requirements are met by all respondents, and that their respective reliability program approaches are deemed "acceptable" by the customer.

The "obvious" winner (although, in reality, these decisions are not

Supplier	Predicted MTBF	Field MTBF	Ratio of Predicted to Observed Field MTBF
A	1845	2174	0.8
B	2000	51	39.2
C	2304	6903	0.3
D	2840	1160	2.4
E	3080	3612	0.9

Table 2 – Ratio of Predicted to Field MTBF for Hypothetical Example

always obvious) is Supplier B, since it meets the MTBF requirement at the lowest cost. An important "unknown" in this scenario is the level of risk associated with this decision.

Suppose that the customer had access to (i.e., required) the information provided in Table 2? How might this affect their decision-making process? Assume that the five suppliers' proposed systems had already been fielded on other programs.

The original choice of Supplier B does not look so good now, as its demonstrated field MTBF represents only 2.55% of its predicted value, and only 3.4% of the stated 1500-hour MTBF requirement. The resulting impact on total life cycle cost would significantly offset the fact that Supplier B was the low-cost bidder.

A better decision for this RFP would have been to select Supplier A or E, both of whose systems have demonstrated field MTBFs that are marginally better than their predicted MTBFs, meet the specified MTBF requirement, and whose cost proposals were ranked second and third, respectively. Although Supplier D predicted a MTBF

continued on next page >>>

indicating that the 1500-hour requirement could be met, its actual field MTBF is only 41% of the predicted and 77% of the requirement, representing legitimate technical and total life cycle cost risk. Finally, while Supplier C may look appealing based on its very low predicted/observed ratio of 0.3, questions to be considered are (1) is it significantly overdesigning its system relative to the 1500-hour requirement and (2) would the increased design/development cost be more than offset by cost savings during the Operations and Support phase of the life cycle? The “obvious” decision has now become significantly less obvious.

The metric proposed to support the acquisition of more reliable systems at lower technical and cost risk is “Historical Observed Reliability – Ratio” (HOR-R, pronounced “horror”), and is defined as the ratio of the final pre-test reliability prediction or assessment value and the most recent observed field reliability value for that system. The benefits of this metric are:

- › It provides a quantitative measure for making informed acquisition decisions based on the risks related to proposed reliability program approaches and total life cycle cost impacts associated with potential suppliers.
- › It is independent of the reliability prediction or assessment methodology used. The methodology can be Handbook-based (MIL-HDBK-217, 217Plus, Telcordia, etc.) or PoF-based (assessment of time-to-wearout relative to field-experience wearout times). Consequently, suppliers are not restricted to a standard. For example, Supplier A can generate an initial prediction based on MIL-HDBK-217, but then tailor it using historical experience from other programs to apply adjustment factors that more closely relate the prediction method to the achieved field reliability for those programs.
- › It can be applied to multiple reliability-based requirements (MTBF, MTTF, R(t), A_o, etc.).
- › It supports the collection, analysis and assessment of field reliability data required by DoDI 5000.02 and ANSI/GEIA-STD-0009 to (1) determine root failure causes, modes and mechanisms, (2) validate in-house modeling, simulation and testing results, and (3) assess reliability program impact on system total life cycle cost.

The constraints of the proposed metric are:

- › Its greater focus on historical field reliability performance requires a larger investment by the customer, and potential suppliers, on up-front DFR activities and downstream failure data collection and analysis to root cause. These larger up-front investments should be offset by savings in system total life cycle costs through reduction in long-term Operations and Support costs
- › Both the customer and potential suppliers will need to exercise greater diligence in the preparation and evaluation of RFP responses to ensure that submitted HOR-R data is sufficient, accurate and verifiable.

Example Wording for RFP Section L

The wording that follows is an abbreviated representation of how the HOR-R metrics could be requested within the context of a RFP.

“The following information shall be entered into the table below (Table 3) to provide insight into the bidder’s historical ability to correlate the documented predicted pre-test reliability of their systems/products with the corresponding observed field reliability during actual customer use. A minimum of three (3) systems/products is requested, representing the three most recent systems/products for which observed field reliability has been measured and documented by either the bidder, or the bidders’ customer (preferred). Documentation in support of the predicted (or assessed) and observed reliability of the system(s)/product(s) listed in the table shall be provided upon request. An inability to provide the requested information in the table, or to provide documentation in support of information provided, will not be cause for disqualification of the bidder from the proposal evaluation process. It will, however, be perceived as an increased level of reliability and life cycle cost risk that will be factored into the evaluation of the bidder’s ability to meet the stated reliability requirements of this RFP.

1. Name or Nomenclature of the System/Product
2. Quantified Reliability Requirement (MTBF = “x” hours, R = “y” for mission time “t”, MTTF = “x” hours, other)
3. Initial Reliability Prediction or Assessment Value
4. Reliability Prediction/Assessment Method(s) Used – Describe (e.g., MIL-HDBK-217F Not 2; Tailored MIL-HDBK-217F Not 2 – describe tailoring; 217Plus; Physics-of-Failure; Telcordia; other)
5. Date of Initial Reliability Prediction/Assessment
6. Final Pre-Test Reliability Prediction/Assessment Value
7. Reliability Prediction/Assessment Method(s) Used – Describe (e.g., MIL-HDBK-217F Not 2; Tailored MIL-HDBK-217F Not 2 – describe tailoring; 217Plus; Physics-of-Failure; Telcordia; other)
8. Date of Final Pre-Test Reliability Prediction/Assessment
9. Achieved/Demonstrated Test Reliability Value
10. Type of Test (e.g., Rel Growth, Rel Demo, Rel Qualification, DT, OT, other)
11. Date of Achieved/Demonstrated Test Reliability
12. Observed Field Reliability (Most Recent Measure)
13. Date of Observed Field Reliability
14. Ratio of Final Pre-Test Reliability Prediction/Assessment (6) to Observed Field Reliability (12)
15. If the Ratio of Predicted/Observed Reliability (Block 14) > 1.00, explain discrepancy and corrective action taken, if any, to improve the reliability prediction/assessment methodology used in (Block 7)”

Recommendation: Two new failure classifications to support reliability growth in the design phase

Dr. Larry Crow is internationally recognized for his career-spanning body of work in the development of models that have been used over the years to assess reliability growth. The identification of A-modes (failure modes in design that will not be mitigated) and B-modes (failure modes in design that will be either mitigated immediately-

type BC modes, or delayed – type BD modes) have been defined by Dr. Crow as a means for quantifying reliability growth in the pre-deployment phases of a system [1, 2]. Table 4 is a hypothetical example of how a technique such as a Failure Modes and Effects Analysis (FMEA) can be easily adapted to reflect an engineer’s assignment of A-modes and B-modes. Note that the designation of BC- and BD-modes would not occur until those modes were actually experienced during testing or actual field use.

The DFR process is intended to promote reliability growth earlier in the design phase of the system life cycle, prior to precipitation of failures and decisions regarding A- and B-modes “discovered” during testing. The authors felt that a set of metrics was needed that quantifies the relative effectiveness of DFR analyses, modeling and simulation in identifying and mitigating these failure modes that highlights opportunities for improvement in these processes. To that end, we

recognized an opportunity to define two new failure classifications that could be used to leverage lessons learned from “current” test failures to improve the robustness and design impact of “future” DFR processes and activities prior to those future systems entering the test phase.

The two new proposed failure classifications are:

- › Unanticipated Mode – defined as a failure mode that is discovered during item testing or field use, but was not documented during DFR analyses, modeling and simulation
- › Unexpected Mode – defined as a failure mode that is accounted for, documented and thought to have been effectively eliminated/mitigated as a direct result of DFR analyses, modeling and simulation, but occurs during item testing or field use anyway

(1) System/Product Name or Nomenclature	(2) Quantified Reliability Requirement	Initial Reliability Prediction/ Assessment			Final Pre-Test Reliability Prediction/Assessment			Achieved/Demonstrated Test Reliability			Observed Field Reliability (Most Recent)		
		(3) Value	(4) Method	(5) Date	(6) Value	(7) Method	(8) Date	(9) Value	(10) Test Type	(11) Date	(12) Value	(13) Date	(14) Ratio of (6)/(12)
System #1													
(15): If (14) > 1.00, explain discrepancy and corrective action(s) taken to improve reliability prediction/assessment method(s):													
(1) System/Product Name or Nomenclature	(2) Quantified Reliability Requirement	Initial Reliability Prediction/ Assessment			Final Pre-Test Reliability Prediction/Assessment			Achieved/Demonstrated Test Reliability			Observed Field Reliability (Most Recent)		
		(3) Value	(4) Method	(5) Date	(6) Value	(7) Method	(8) Date	(9) Value	(10) Test Type	(11) Date	(12) Value	(13) Date	(14) Ratio of (6)/(12)
System #2													
(15): If (14) > 1.00, explain discrepancy and corrective action(s) taken to improve reliability prediction/assessment method(s):													

Notes: (Provide any additional details for Blocks (3) through (14) – identify comments by System # and Block #)

Table 3 – Suggested Template for RFP Section L Reliability Prediction Requirements

Index No.	Unit	Function	Failure Mode	Possible Failure Causes	Effect On...			Fail. Rate	O	S	D	RPN	Failure Mode Type
					Unit	Sub	Sys						
1.1	Range Output	Outputs range data to user display	Data value is high vs. actual range	Logic problem; computation problem; data handling problem	N/A	None	Mission Degraded	0.008	6	10	4	240	B
1.2			Data value is low vs. actual range	Logic problem; computation problem; data handling problem	N/A	None	Mission Degraded	0.008	6	10	4	240	B
1.3			Output data not sent to display	Logic problem; interface/timing fault	N/A	None	Mission Aborted	0.001	4	10	1	40	B
1.4			Range output fluctuates within specs	Data handling problem	N/A	None	No Mission Impact	X	X	X	X	X	X

Table 4 – Hypothetical FMEA Form Modified to Reflect Failure Mode Types

continued on next page >>>

Expanding these definitions to the Crow Extended Reliability Growth Model [2] yields:

- › $A_{unanticipated}$ Mode – a failure mode that is discovered during item testing or field use that had not been documented during DFR activities. No corrective action is taken, but DFR analyses are updated.
- › $A_{unexpected}$ Mode – a failure mode that is documented and thought to have been effectively mitigated as a direct result of DFR activities, but occurs during item testing or field use anyway. No corrective action is taken.
- › $BC_{unanticipated}$ Mode – a failure mode that is discovered during item testing or field use that had not been documented during DFR activities. Corrective action is taken immediately and DFR analyses are updated.
- › $BC_{unexpected}$ Mode – a failure mode that is documented and thought to have been effectively mitigated as a direct result of DFR activities, but occurs during item testing or field use anyway. Corrective action is taken immediately and DFR analyses are updated.
- › $BD_{unanticipated}$ Mode – a failure mode that is discovered during item testing or field use that had not been documented during DFR activities. Corrective action is delayed until test completion or a designated cut-in date, and DFR analyses are updated.
- › $BD_{unexpected}$ Mode – a failure mode that is documented and thought to have been effectively mitigated as a direct result of previous DFR activities, but occurs during item testing or field use anyway. Corrective action is delayed until test completion or a designated cut-in date, and DFR analyses are updated.

Table 5 is adapted from [2], modified to reflect three BC failure modes, and to show how the new failure classifications could be translated to the Crow Extended Reliability Growth Model.

While space in the current paper and presentation does not permit it, Reference [3] expands the 33 metrics of the Crow Extended Reliability Growth Model [2] to include the above definitions, and presents a hypothetical example of the impact of these new definitions on the attained results, such that the need to improve DFR analyses, modeling and simulation for the “next” system, and the level of improvement achieved over time, can be quantified.

There are two general conclusions that can be drawn from the new metrics presented here. In both cases, the improvements to DFR processes are a relative quantitative measure that is specific to each company. A baseline must first be established for the developer’s practices to assess how robust its current DFR process is (i.e., the initial unanticipated- and unexpected-based metrics to be used for the current system). Using these results, the developer would then objectively evaluate areas for DFR process improvement and implement the necessary corrective actions so that these “corrections” will be quantitatively reflected in future designs (measured by higher initial system reliability prior to entering the test phase).

j	X_j	Mode	Category	Corrective Action
1	15.0	B1	$BD_{unexpected}$	Redesign needed. CA delayed.
2	25.3	B2	$BD_{unanticipated}$	FMECA updated. CA delayed.
3	47.5	B3	$BD_{unexpected}$	Redesign needed. CA delayed.
4	54.0	B4	$BD_{unanticipated}$	FMECA updated. CA delayed.
5	56.4	B5	$BD_{unexpected}$	Redesign needed. CA delayed.
6	63.6	A	$A_{unanticipated}$	FMECA updated. No CA.
7	72.2	B5	Expected	
8	99.6	B6	$BD_{unexpected}$	Redesign needed. CA delayed.
9	100.3	B7	$BD_{unanticipated}$	FMECA updated. CA delayed.
10	102.5	A	Expected	
11	112.0	B8	$BD_{unanticipated}$	FMECA updated. CA delayed.
12	120.9	B2	Expected	
13	125.5	B9	$BD_{unanticipated}$	FMECA updated. CA delayed.
14	133.4	B10	$BD_{unexpected}$	Redesign needed. CA delayed.
15	164.7	B9	Expected	
16	177.4	B10	Expected	
17	192.7	B11	$BD_{unexpected}$	Redesign needed. CA delayed.
18	213.0	A	$A_{unanticipated}$	FMECA updated. No CA.
19	244.8	A	$A_{unexpected}$	Re-evaluate FMECA for potential change to B-mode.
20	249.0	B12	$BD_{unexpected}$	Redesign needed. CA delayed.
21	250.8	A	Expected	
22	260.1	B1	Expected	
23	263.5	B8	Expected	
24	273.1	A	$A_{unanticipated}$	FMECA updated. No CA.
25	274.7	B6	Expected	
26	285.0	B13	$BD_{unanticipated}$	FMECA updated. CA delayed.
27	304.0	B9	Expected	
28	315.4	B4	Expected	
29	317.1	A	Expected	
30	320.6	A	Expected	
31	324.5	B12	Expected	
32	324.9	B10	Expected	
33	342.0	B5	Expected	
34	350.2	B3	Expected	
35	364.6	B10	Expected	
36	364.9	A	$A_{unanticipated}$	FMECA Updated. No CA necessary.
37	366.3	B2	Expected	
38	373.0	B8	Expected	
39	379.4	B14	$BC_{unanticipated}$	FMECA updated. CA immediate.
40	389.0	B15	$BC_{unexpected}$	Redesign needed. CA immediate.
41	394.9	A	Expected	
42	395.2	B16	$BC_{unexpected}$	Redesign needed. CA immediate.

Table 5 – Modified Test Data Table from [2]

The first conclusion is that the larger the percent contribution of unanticipated A-, BC- and BD-modes to the total number of A-, BC- and BD-modes, the less robust the DFR process is in proactively identifying failure modes prior to testing. The corrective action to the DFR process is predicated on an evaluation of current DFR

analyses, modeling and simulation techniques and tools to improve their ability to identify and document failure modes.

The second conclusion is that the larger the percent contribution of unexpected A-, BC- and BD-modes to the total number of A-, BC- and BD-modes, the less effective the DFR process is in mitigating known failure modes through redesign or other techniques currently being used by the developer. The corrective action in this case is evaluation and improvement of the developer's reliability design practices, rules, procedures, etc., in order to more effectively mitigate failure modes that are already documented prior to testing.

Conclusions

This paper has presented two recommendations for improving the acquisition and growth of reliable systems that support the intent of DoDI 5000.02 and ANSI/GEIA-STD-0009

The first recommendation is to use a metric based on an experience ratio of a potential supplier's predicted or assessed system reliability measure to its observed field system reliability measure (HOR-R) as a means for selecting suppliers and evaluating reliability and total life cycle cost risk during the proposal evaluation and selection process. Consistent HOR-R metric values less than or equal to 1.0 provide confidence that the supplier has a repeatable process for translating its prediction methodology of choice into correlated field experience that meets or is better than the reliability requirement, with limited reliability and life cycle cost risk to the program. HOR-R metric values greater than 1.0 indicate increasing potential risk to the customer, in that the supplier has not demonstrated an ability to achieve reliability requirements in the field based on its prediction or assessment techniques. This implies increased life cycle cost risk. The inability of a supplier to provide any HOR-R metric value represents an unknown level of reliability and life cycle cost risk to the program.

Significant advantages to this metric are:

- › Any reliability prediction or assessment technique such as standard or tailored empirical handbooks, PoF, etc., can be used, since the metric effectiveness is based on "real-world" experience, not the ability of the chosen technique to generate a "compliant" number.
- › The metric can be effectively applied to different quantitative reliability requirements (MTBF, MTTF, $R(t)$, A_o , etc.)

The second recommendation is to extend the definition of reliability A-mode and B-mode failures to include classifications of "Unanticipated Failure Mode" and "Unexpected Failure Mode" to establish relative metrics that drive improvements in DFR analyses, modeling and simulation processes. The larger the percent contribution of unanticipated failure modes to total failure modes, the less robust the supplier's DFR process is for proactively identifying failure

modes prior to entering the test phase. Corrective action is based on the evaluation of current DFR analyses, modeling and simulation processes to improve their ability to identify and document failure modes. The larger the percent contribution of unexpected failure modes to total failure modes, the less effective the supplier's DFR process is in mitigating previously identified failure modes through redesign or other mitigation techniques. Corrective action is implemented to improve reliability design practices, rules, procedures, etc., to more effectively mitigate known failure modes prior to entering the test phase.

These two recommendations, coupled with the corrective actions they initiate, provide measurable benchmarks to improve both the effectiveness of acquisitions in becoming more aware of high-risk decisions, and the ability of DFR activities to proactively mitigate failure modes prior to their more costly discovery during testing or field use.

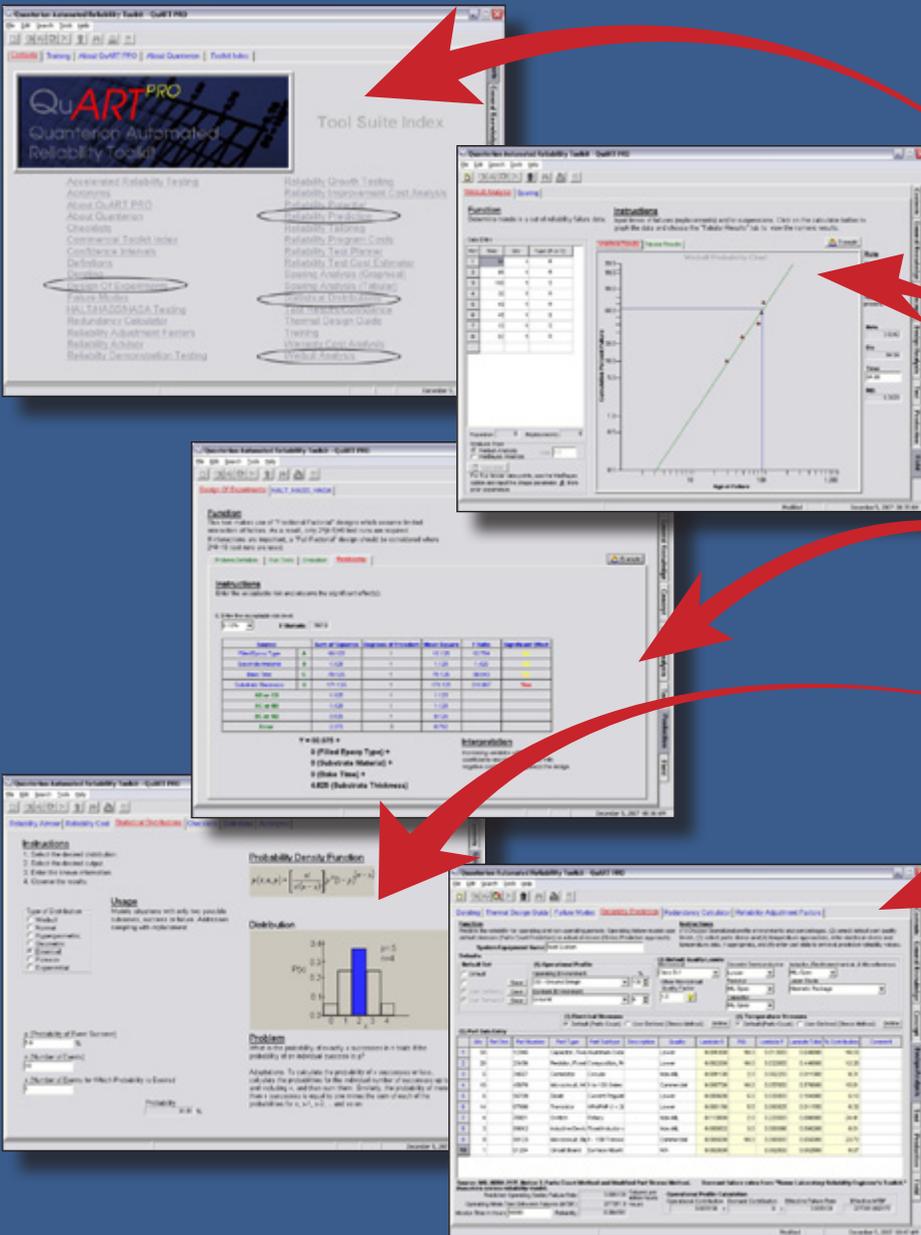
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References

1. L.H. Crow, "Achieving High Reliability", Journal of the Reliability Analysis Center, Vol. 4, 2000
2. L.H. Crow, "An Extended Reliability Growth Model for Managing and Assessing Corrective Actions", Proceedings 2004 Annual Reliability and Maintainability Symposium, January 2004, pp. 73-80
3. DoDI 5000.02, "Operation of the Defense Acquisition System", 2 December 2008
4. ANSI/GEIA-STD-0009-2008, "Reliability Program Standard for Systems Design, Development, and Manufacturing", 13 November 2008,
5. D.B. Nicholls and P. Lein, "Introducing New Failure Classifications into the Crow Extended Reliability Growth Model", Journal of the Reliability Information Analysis Center, Vol. 17, No. 3, September 2009

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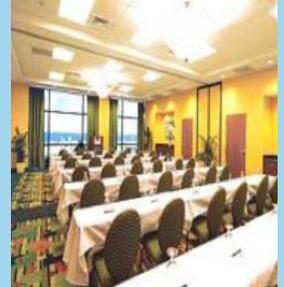
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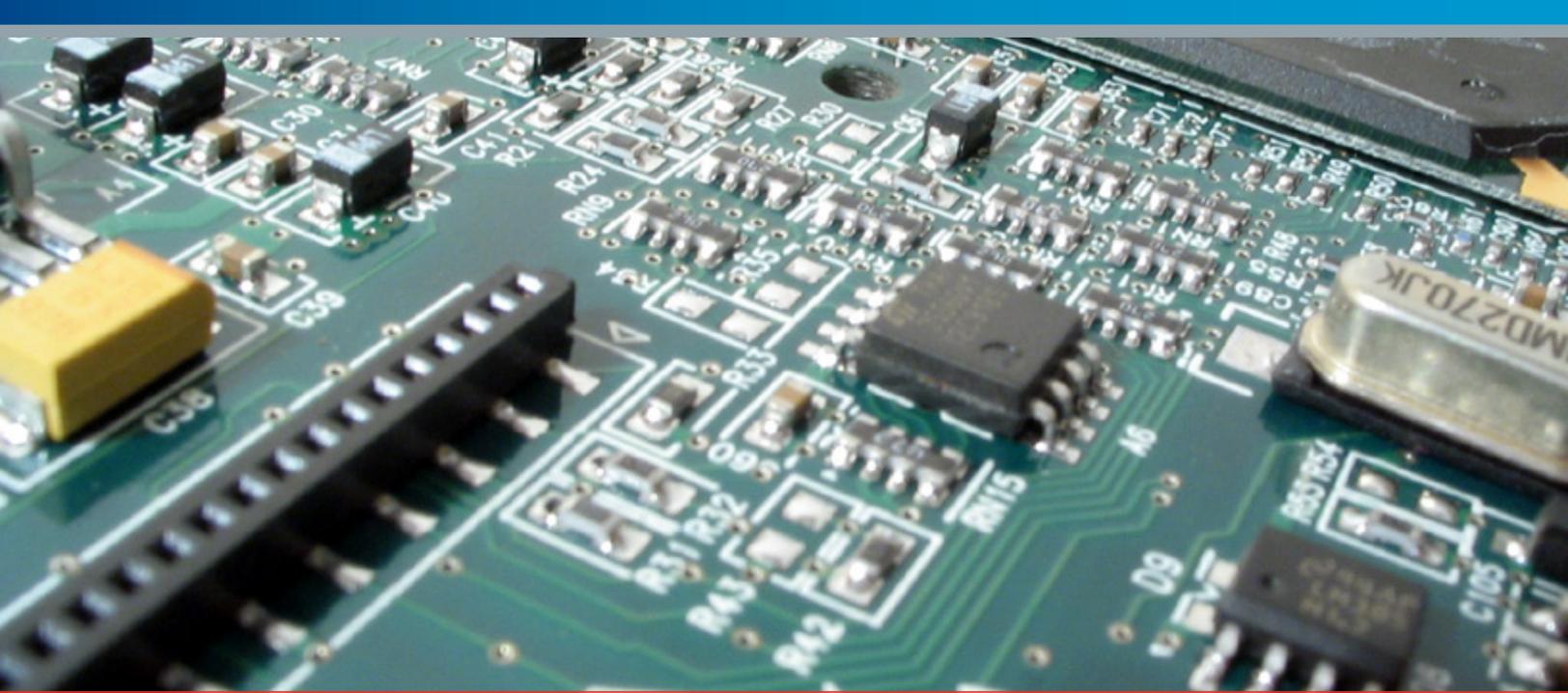
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INTRODUCING UNANTICIPATED AND UNEXPECTED FAILURES TO THE CROW EXTENDED CONTINUOUS EVALUATION RELIABILITY GROWTH MODEL

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In our papers entitled “Two Recommendations for the Acquisition and Growth of Reliable Systems” [Reference 1] and “Improving Design for Reliability (DFR) Processes Using Modified Crow Extended Reliability Growth Model Metrics” [Reference 2], we introduced a recommendation for defining two new failure mode classifications:

- **Unanticipated Mode** – A failure mode that is discovered during item testing or field use, but was not documented during DFR analyses, modeling and simulation
- **Unexpected Mode** – A failure mode that is accounted for, documented and thought to have been effectively eliminated/mitigated as a direct result of DFR analyses, modeling and simulation, but occurs during item testing or field use anyway

Two straightforward metrics were proposed to quantitatively measure the relative effectiveness of the overall DFR process:

$$\% \text{ of Unanticipated Failures} = \frac{\# \text{ of Unanticipated Failures in Test or Field}}{\# \text{ of Total Failures in Test or Field}}$$

$$\% \text{ of Unexpected Failures} = \frac{\# \text{ of Unexpected Failures in Test or Field}}{\# \text{ of Unexpected Failures in Test or Field}}$$

As stated in Reference 2, the obvious goal is to drive each of these metrics to zero, and the measure of success from one system to the next is the ability to reduce the value of these metrics for each successive system.

The Reference 2 paper also extended the concept of unanticipated and unexpected failure modes to Dr. Larry Crow’s Extended Reliability Growth Model [Reference 3] by incorporating the new definitions into Equations 17 and 22 and each of the 33 reliability growth management metrics discussed in his paper. At that time, Dr. Crow’s definitions of failure mode types were:

Type A:	Failure modes that, if seen, are not corrected
Type BC:	Failure mode that, if seen, is always corrected during test (corrective action immediate or occurs before test is completed)
Type BD:	Failure mode that, if seen, is always corrected after all testing has been completed (corrective action delayed)

At the 2010 Annual Reliability and Maintainability Symposium (RAMS), Dr. Crow introduced an enhancement to his basic model form. In his paper, “The Extended Continuous Evaluation Reliability Growth Model” [Reference 4], he introduced a revised set of metrics that allows for continuous evaluation and management of the “reliability growth of a system across multiple test phases and to accommodate failures that are likely to be seen during “Operational-Like” testing.”

To accommodate this new model, Dr. Crow redefined and enhanced

his original failure mode types to the following:

Type A:	Same as Crow basic Extended Model definition
Type BC:	Failure mode that, if seen, receives corrective action immediately at the time of failure, before testing continues
Type BD:	Failure mode that, if seen, receives corrective action at some time <u>after</u> the first occurrence of that failure mode
Type BDC:	Type BD failure mode that has been corrected at some time before the test has ended (delayed, but corrected during the test)
Type BDD:	Type BD failure mode that has not been corrected at the time the test has ended (delayed, but not corrected during the test)

The objective of our current paper, then, is to apply our two new failure mode classifications to the new or modified metrics of Dr. Crow's latest model. The reader is encouraged to review References 1 through 4 to thoroughly understand the background associated with our approach. In that context, this paper will only cover the modified metrics, and their associated equations, from Reference 4. Specifically, only those metrics from References 2 and 3 which are impacted by the new definitions of Type BDC and Type BDD failure modes will be covered

From Reference 4, Dr. Crow has stated that each time an assessment of system reliability is made using the Extended Continuous Evaluation Model, the following metrics can be calculated:

- › Current Demonstrated MTBF
- › Nominal Growth Potential
- › Nominal Average Effectiveness Factor (EF)
- › Nominal Projection if BDD modes are corrected with Nominal EFs
- › Actual Growth Potential
- › Actual Average EF
- › Actual Projection if BDD modes are corrected with Actual EFs
- › Rate of Discovery

In the modified metrics that follow, we will relate each metric to the equation number from Dr. Crow's Reference 4 paper (Eq. #).

Current Demonstrated MTBF

The current demonstrated MTBF is given in Reference 4 based on the Crow (AMSAA) model demonstrated failure intensity (Eq. 3), the associated Weibull slope parameter (Eq 4) and the estimated scale parameter for the Crow (AMSAA) Model (Eq. 5). With the new failure mode classifications, this estimated scale parameter becomes:

$$\hat{\lambda} = \frac{(N_{unanticipated} + N_{unexpected} + N_{expected})}{T}$$

where,

$$\hat{\lambda}_{unanticipated} = \frac{N_{unanticipated}}{T}$$

$$\hat{\lambda}_{unexpected} = \frac{N_{unexpected}}{T}$$

$$\hat{\lambda}_{expected} = \frac{N_{expected}}{T}$$

The Current Demonstrated MTBF is simply the inverse of the demonstrated failure intensity (Eq. 9).

Nominal Growth Potential, Nominal Average Effectiveness Factor (EF), Rate of Discovery and Nominal Projection

The Crow Nominal Growth Potential Factor (Eq. 10) becomes:

$$\lambda_{NGPFactor} = \lambda_{NGPFactor-unanticipated} + \lambda_{NGPFactor-unexpected} + \lambda_{NGPFactor-expected}$$

where,

$$\lambda_{NGPFactor-unanticipated} = \sum_{i=1}^{K_{BDD-unanticipated}} (1 - d_i^{NomBDD-unanticipated}) \left(\frac{N_i}{T} \right)$$

$$\lambda_{NGPFactor-unexpected} = \sum_{i=1}^{K_{BDD-unexpected}} (1 - d_i^{NomBDD-unexpected}) \left(\frac{N_i}{T} \right)$$

$$\lambda_{NGPFactor-expected} = \sum_{i=1}^{K_{BDD-expected}} (1 - d_i^{NomBDD-expected}) \left(\frac{N_i}{T} \right)$$

The determination of the assigned nominal average effectiveness factor, $d_i^{NomBDD-x}$, is based on the appropriate Type BDD failure mode status, i.e., $BDD_{unanticipated}$, $BDD_{unexpected}$ and $BDD_{expected}$.

The equation for the probability of Type BDD failures at time "T" is based on the total number of distinct Type BDD modes at "T" divided by the sum of the total number of distinct Type BDD and BDC modes at "T". Using our "unanticipated" and "unexpected" categories, the equations become:

$$p(T) = p(T)_{unanticipated} + p(T)_{unexpected} + p(T)_{expected}$$

where,

$$p(T)_{unanticipated} = \frac{\# \text{ of distinct BDD}_{unanticipated} \text{ modes at T}}{\left(\# \text{ of distinct BDD}_{unanticipated} \text{ modes at T} \right) + \left(\# \text{ of distinct BDC}_{unanticipated} \text{ modes at T} \right)}$$

$$p(T)_{unexpected} = \frac{\# \text{ of distinct BDD}_{unexpected} \text{ modes at T}}{\left(\# \text{ of distinct BDD}_{unexpected} \text{ modes at T} \right) + \left(\# \text{ of distinct BDC}_{unexpected} \text{ modes at T} \right)}$$

$$p(T)_{expected} = \frac{\# \text{ of distinct BDD}_{expected} \text{ modes at T}}{\left(\# \text{ of distinct BDD}_{expected} \text{ modes at T} \right) + \left(\# \text{ of distinct BDC}_{expected} \text{ modes at T} \right)}$$

continued on next page >>>

The equation for the Type BDD mode failure intensity (Eq 12) becomes:

$$\lambda_{BDD} = \lambda_{BDD-unanticipated} + \lambda_{BDD-unexpected} + \lambda_{BDD-expected}$$

where,

$$\hat{\lambda}_{BDD-unanticipated} = \frac{N_{BDD-unanticipated}}{T}$$

$$\hat{\lambda}_{BDD-unexpected} = \frac{N_{BDD-unexpected}}{T}$$

$$\hat{\lambda}_{BDD-expected} = \frac{N_{BDD-expected}}{T}$$

As stated by Dr. Crow (Ref. 4), the discovery function (or rate of discovery) represents the rate at which new, distinct Type BD modes are discovered during the test. It is calculated using all first occurrences of the total number of Type BD modes (including Types BDC and BDD). In this equation, the variable "M" represents the count of all unique Type BD modes, and Z_i corresponds to the time at which each unique Type BDC and Type BDD mode is discovered during the test. Using these variables in the equation for the unbiased estimate of beta (Eq. 14) for the h(t) function, and incorporating our two failure classifications, yields:

$$\beta_{unanticipated}^* = \frac{(M_{BD-unanticipated} - 1)}{\sum_{i=1}^{M_{BD-unanticipated}} \ln\left(\frac{T}{Z_{iBD-unanticipated}}\right)}$$

$$\beta_{unexpected}^* = \frac{(M_{BD-unexpected} - 1)}{\sum_{i=1}^{M_{BD-unexpected}} \ln\left(\frac{T}{Z_{iBD-unexpected}}\right)}$$

$$\beta_{expected}^* = \frac{(M_{BD-expected} - 1)}{\sum_{i=1}^{M_{BD-expected}} \ln\left(\frac{T}{Z_{iBD-expected}}\right)}$$

The h(T) function equation (Eq. 15) then becomes:

$$h(T) = h(T)_{unanticipated} + h(T)_{unexpected} + h(T)_{expected}$$

where,

$$h(T)_{unanticipated} = \beta_{unanticipated}^* \left(\frac{M_{BD-unanticipated}}{T}\right)$$

$$h(T)_{unexpected} = \beta_{unexpected}^* \left(\frac{M_{BD-unexpected}}{T}\right)$$

$$h(T)_{expected} = \beta_{expected}^* \left(\frac{M_{BD-expected}}{T}\right)$$

The Nominal Growth Potential failure intensity (Eq. 16) becomes:

$$\lambda_{NGP} = \lambda_{NGP-unanticipated} + \lambda_{NGP-unexpected} + \lambda_{NGP-expected}$$

where,

$$\lambda_{NGP-unanticipated} = \lambda_{D-unanticipated} - \lambda_{BDD-unanticipated} + \lambda_{NGPFactor-unanticipated} + \left(d_{NomBDD-unanticipated} * p(T)_{unanticipated} * h(T)_{unanticipated}\right) - \left(d_{NomBDD-unanticipated} * h(T)_{unanticipated}\right)$$

$$\lambda_{NGP-unexpected} = \lambda_{D-unexpected} - \lambda_{BDD-unexpected} + \lambda_{NGPFactor-unexpected} + \left(d_{NomBDD-unexpected} * p(T)_{unexpected} * h(T)_{unexpected}\right) - \left(d_{NomBDD-unexpected} * h(T)_{unexpected}\right)$$

$$\lambda_{NGP-expected} = \lambda_{D-expected} - \lambda_{BDD-expected} + \lambda_{NGPFactor-expected} + \left(d_{NomBDD-expected} * p(T)_{expected} * h(T)_{expected}\right) - \left(d_{NomBDD-expected} * h(T)_{expected}\right)$$

The individual Nominal Growth Potential MTBFs are simply the inverse of their respective Nominal Growth Potential failure intensities (Eq. 18).

Dr. Crow defines the Nominal Projection metric as an estimation of the failure intensity (Eq. 20) and MTBF (Eq. 22) if all seen Type BDD failure modes are corrected at time "T". The modified failure intensity Nominal Projection equation becomes:

$$\lambda_{NP} = \lambda_{NP-unanticipated} + \lambda_{NP-unexpected} + \lambda_{NP-expected}$$

where,

$$\lambda_{NP-unanticipated} = \lambda_{NGP-unanticipated} + \left(d_{NomBDD-unanticipated} * h(T)_{unanticipated}\right)$$

$$\lambda_{NP-unexpected} = \lambda_{NGP-unexpected} + \left(d_{NomBDD-unexpected} * h(T)_{unexpected}\right)$$

$$\lambda_{NP-expected} = \lambda_{NGP-expected} + \left(d_{NomBDD-expected} * h(T)_{expected}\right)$$

The Nominal Projection MTBFs are simply the inverse of their respective Nominal Projection failure intensities (Eq. 18).

Actual Growth Potential, Actual Average Effectiveness Factor (EF) and Actual Projection

As indicated by Dr. Crow (Ref. 4), the Nominal metrics are all based on a presumption that all Type BDD failure modes have been fixed by time "T". If only a subset of the Type BDD modes are fixed by time "T", however, the Actual metrics from Dr. Crow's paper need to be

used to gain more accurate insight into the interim reliability growth characteristics of the system during the test.

The Actual Growth Potential Factor (Eq. 24), modified to reflect unanticipated and unexpected failures, is calculated as:

$$\lambda_{AGPFactor} = \lambda_{AGPFactor-unanticipated} + \lambda_{AGPFactor-unexpected} + \lambda_{AGPFactor-expected}$$

where,

$$\lambda_{AGPFactor-unanticipated} = \sum_{j=1}^{K_{BDD-unanticipated}} \left(1 - d_i^{ActBDD-unanticipated}\right) \left(\frac{N_j}{T}\right)$$

$$\lambda_{AGPFactor-unexpected} = \sum_{j=1}^{K_{BDD-unexpected}} \left(1 - d_i^{ActBDD-unexpected}\right) \left(\frac{N_j}{T}\right)$$

$$\lambda_{AGPFactor-expected} = \sum_{j=1}^{K_{BDD-expected}} \left(1 - d_i^{ActBDD-expected}\right) \left(\frac{N_j}{T}\right)$$

The Actual Growth Potential failure intensity equation (Eq. 26) becomes:

$$\lambda_{AGP} = \lambda_{AGP-unanticipated} + \lambda_{AGP-unexpected} + \lambda_{AGP-expected}$$

where,

$$\lambda_{AGP-unanticipated} = \lambda_{D-unanticipated} - \lambda_{BDD-unanticipated} + \lambda_{AGPFactor-unanticipated} + \left(d_{ActBDD-unanticipated} * p(T)_{unanticipated} * h(T)_{unanticipated}\right) - \left(d_{ActBDD-unanticipated} * h(T)_{unanticipated}\right)$$

$$\lambda_{AGP-unexpected} = \lambda_{D-unexpected} - \lambda_{BDD-unexpected} + \lambda_{AGPFactor-unexpected} + \left(d_{ActBDD-unexpected} * p(T)_{unexpected} * h(T)_{unexpected}\right) - \left(d_{ActBDD-unexpected} * h(T)_{unexpected}\right)$$

$$\lambda_{AGP-expected} = \lambda_{D-expected} - \lambda_{BDD-expected} + \lambda_{AGPFactor-expected} + \left(d_{ActBDD-expected} * p(T)_{expected} * h(T)_{expected}\right) - \left(d_{ActBDD-expected} * h(T)_{expected}\right)$$

As before, the Actual Growth Potential MTBFs are simply the inverse of their respective Nominal Projection failure intensities (Eq. 28).

The Actual Project Growth failure intensity at time "T" (Eq. 30) is modified to become:

$$\lambda_{AP} = \lambda_{AP-unanticipated} + \lambda_{AP-unexpected} + \lambda_{AP-expected}$$

where,

$$\lambda_{AP-unanticipated} = \lambda_{AGP-unanticipated} + \left(d_{ActBDD-unanticipated} * h(T)_{unanticipated}\right)$$

$$\lambda_{AP-unexpected} = \lambda_{AGP-unexpected} + \left(d_{ActBDD-unexpected} * h(T)_{unexpected}\right)$$

$$\lambda_{AP-expected} = \lambda_{AGP-expected} + \left(d_{ActBDD-expected} * h(T)_{expected}\right)$$

The Actual Projected MTBFs at time "T" are the inverse of their respective Actual Projection failure intensities (Eq. 32).

Conclusions

As stated in our original RIAC Journal article (Reference 2), the results from these new metrics provide useful insight into the effectiveness of DFR processes in detecting (i.e., anticipating) and mitigating (i.e., not expecting them to occur) failure modes. Their initial application is for establishing a baseline measure within "your" specific company to quantify how robust your DFR processes and corrective actions are. Although your first system may not benefit from the results (since your initial set of unanticipated and unexpected failures are going to be, unfortunately, discovered during test), the resulting corrective actions to your DFR processes and design mitigation approaches should result in quantifiable improvement in initial system reliability preceding any formal reliability testing.

References

1. D.B. Nicholls and P. Lein, "Two Recommendations for the Acquisition and Growth of Reliable Systems", Proceedings of the 2010 Annual Reliability and Maintainability Symposium, January 2010
2. D.B. Nicholls and P. Lein, "Improving Design for Reliability (DFR) Processes Using Modified Crow Extended Reliability Growth Model Metrics", Journal of the Reliability Information Analysis Center, Vol. 17, No. 3, September 2009
3. L.H. Crow, "An Extended Reliability Growth Model for Managing and Assessing Corrective Actions", Proceedings of the 2004 Annual Reliability and Maintainability Symposium, January 2004, pp. 73-80
4. L.H. Crow, "The Extended Continuous Evaluation Reliability Growth Model", Proceedings of the 2010 Annual Reliability and Maintainability Symposium, January 2010

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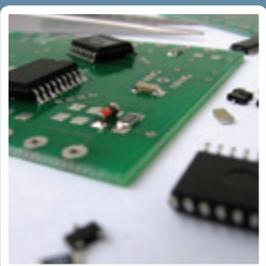
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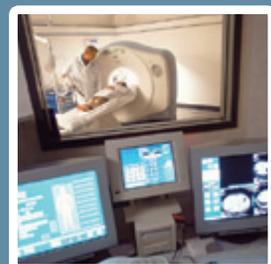
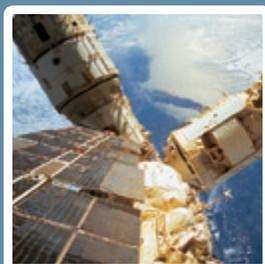
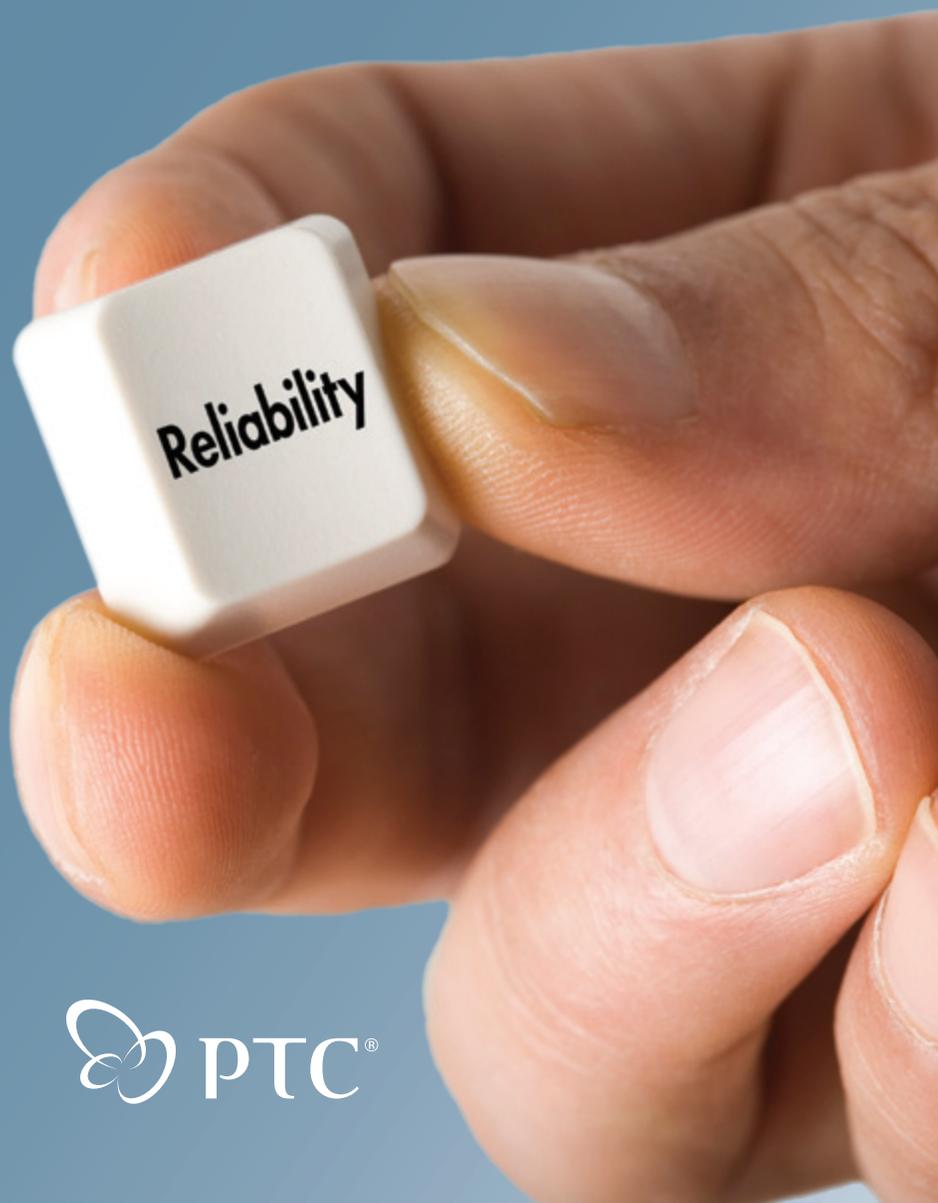
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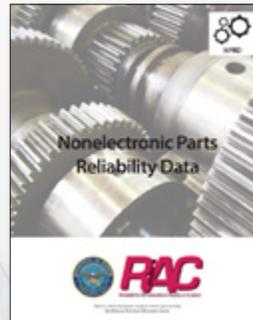
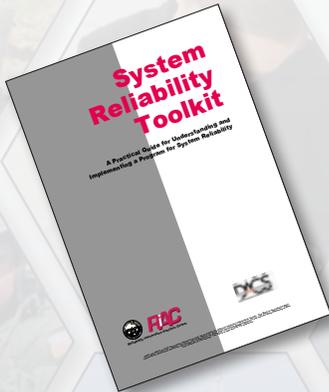
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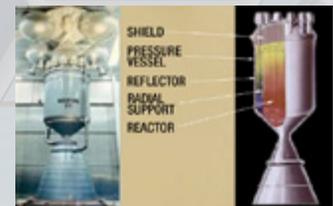


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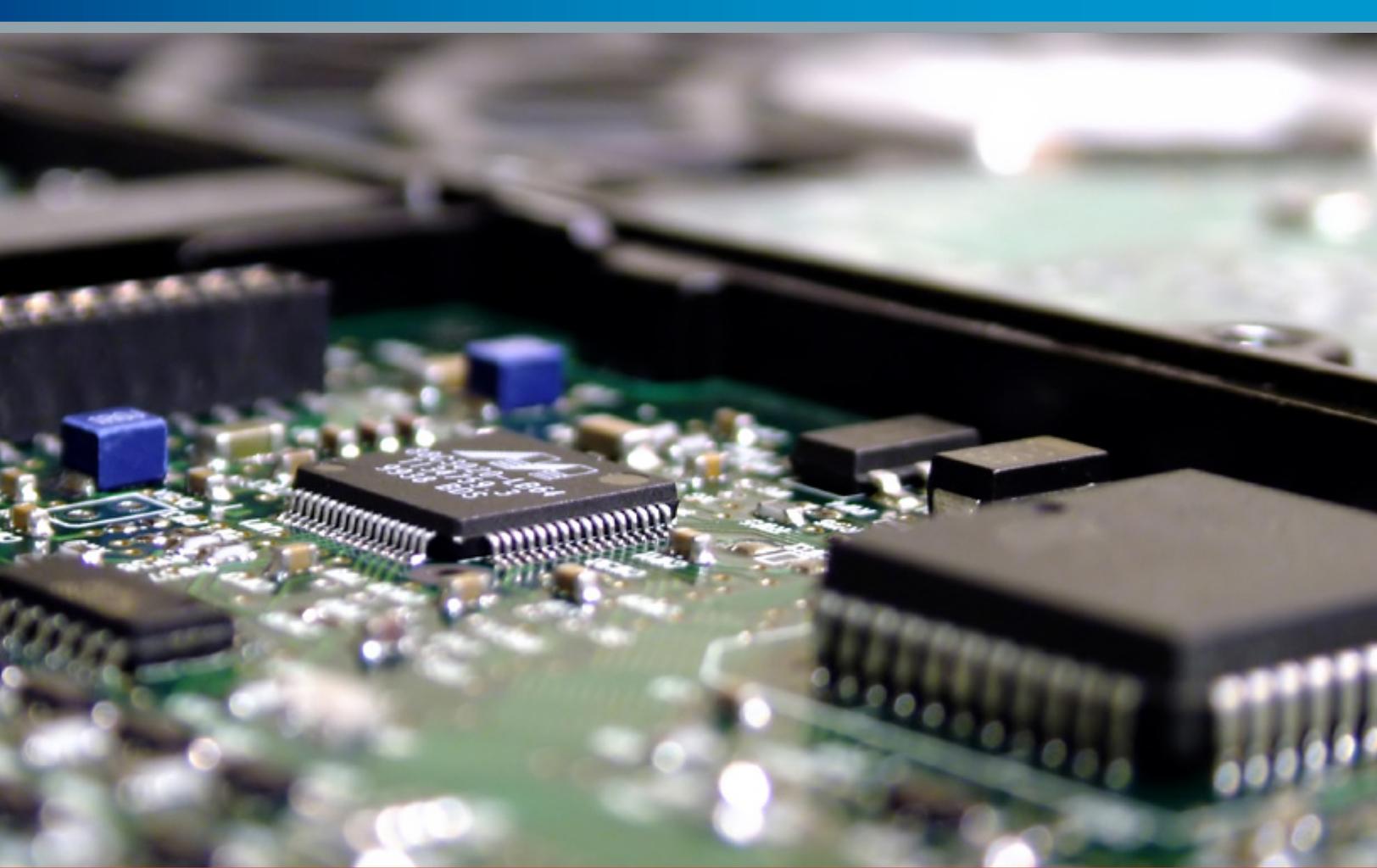
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MEASURING FAILURE RATE BY TESTING

Vito Faraci Jr

Introduction

Most leading IC manufacturers nowadays take measuring failure rates of their IC components very seriously. They spend a lot of time, money, and energy performing various kinds of lab testing to accomplish this. One such test scenario requires placing a given number of components (ICs) in an oven, and “baking” them for a predetermined amount of time. After which, the ICs are tested to determine how many of them failed in the process. The following is an algorithm (equation) excerpted from one of the world’s leading IC manufacturers National Semiconductor. It should be stated that this algorithm is an industry standard for other leading manufacturers as well. As can be seen, it utilizes a Chi Square (χ^2) Distribution Table to calculate maximum failure rate λ_{MAX} . The quantitative inputs to this algorithm is the number of devices (ICs) being tested, the number of hours (under test), the number of failures detected, and α (confidence level in percent). The output is the maximum failure rate (minimum MTF) of the IC associated with the specified confidence level α .

Excerpt from National Semiconductor

$$\lambda_{MAX} = \frac{\chi_{1-\alpha}^2 [with\ df = 2(r + 1)]}{2T}$$

Maximum Failure Rate or worst case where:

χ^2 = Chi Square Distribution

r = Number of Failures

df = Degrees of freedom

T = Total number error test hours (number of devices x number of hours)

α = Statistical error expected in estimate. For 60% confidence level, $\alpha = 0.6$

Alpha can then be interpreted to mean that we can state with statistical confidence level of alpha (i.e., 60%) that the actual failure rate is equal to or less than the calculated maximum (λ_{MAX}) failure rate.

It is interesting to note that even though this “Lambda” algorithm is an industry standard that has been around since the 1950s, and is documented in various Mil-Handbooks, the question of how and why it works is not well documented. Most books or articles on this subject deal with the “How to”. They will explain how to use the algorithm to calculate λ_{MAX} but will not explain the how and why it works. No models or concepts, just information on how to use the algorithm to calculate required results. It will probably be no surprise to anyone that the foundation of this algorithm is rooted in pure probability theory as this paper will show.

Objectives

This paper will attempt to:

1. derive the above algorithm, and
2. explain the how and why it works, in common language, without the use of complicated statistical analyses.

Required (Need to Know) Topics

In order to achieve the above objectives, a familiarization of the following topics is required:

- A) Common problem taken from Reliability involving “n” components with identical failure rates operating active redundant (typical situation when a manufacturer is testing a batch of components),
- B) Definition of Probability Density Function (pdf),
- C) Reliability pdf,
- D) The Chi Square Table,
- E) Poisson Approximation Theorem (See Appendix).

A) Common Problem taken from Reliability:

Three identical black boxes (components with equal failure rate) are placed into operation at the same time (active redundant). What is the probability that at least two black boxes will operate if the reliability (probability of success) of each box is 0.9?

Solution:

Let $p = 0.9$ (probability of success of each box) then $q = 1 - p = 0.1$ (probability of failure of each box). The probabilities of exactly 3, 2, 1, and 0 operating (or 0, 1, 2, and 3 failures) can be easily computed using the following elementary logical procedure.

$$\begin{aligned}
 1 &= (p + q) \\
 1 &= (p + q)^3 \\
 1 &= p^3 + 3p^2q + 3pq^2 + q^3 \\
 1 &= (.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + (.1)^3
 \end{aligned}$$

Or looking at it another way,

$$\begin{aligned}
 1 &= (p + q) \\
 1 &= (p + q)^3 \\
 1 &= p^3 + 3p^2q + 3pq^2 + q^3 \\
 1 &= (.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + (.1)^3
 \end{aligned}$$

$$\text{So } P(\text{at least 2 boxes operating}) = P(3 \text{ or } 2 \text{ boxes operating}) = (.9)^3 + 3(.9)^2(.1) = 0.972$$

A) Same Problem Generalized:

“n” identical components are placed into operation at the same time (active redundant). What is the probability that r or less failures occur if the reliability (probability of success) of each box is p?

Solution:

Let $p =$ probability of success, then $q = 1 - p =$ probability of failure. The probabilities of exactly (n, n-1, ..., n-r, ..., 1, and 0 operating), or said in another way (0, 1, ..., r, ..., n-1, and n failures) can be calculated by generating the binomial expansion as follows:

$$\begin{aligned}
 1 &= (p + q) \\
 1 &= (p + q)^n
 \end{aligned}$$

$$1 = p^n + np^{n-1}q + \dots + \frac{n!}{(n-r)!r!} p^{n-r} q^r + \dots + npq^{n-1} + q^n \quad (\text{expansion})$$

Therefore, $P(r \text{ or less failures})$ is equal to the sum of the first $r + 1$ terms of the above binomial expansion. Stated as a summation: $P(r \text{ or less failures})$

$$= \sum_{k=0}^r \left(\frac{n!}{k!(n-k)!} \right) p^{n-k} q^k \quad (1)$$

This binomial expansion gets somewhat difficult to handle when n gets large. In an effort to make the mathematics easier to handle for large n, the famous Poisson Approximation Theorem is utilized. The theorem essentially states that if n is large and q is small, the following approximation is very accurate for any k.

$$\frac{n!}{k!(n-k)!} p^{n-k} q^k \approx \frac{(nq)^k}{k!} e^{-nq} \quad (\text{See Appendix for a}$$

proof of this.) Therefore, $P(r \text{ or less failures})$

$$\approx \sum_{k=0}^r \frac{(nq)^k}{k!} e^{-nq} = e^{-nq} \left(1 + nq + \frac{(nq)^2}{2!} + \dots + \frac{(nq)^{r-1}}{(r-1)!} + \frac{(nq)^r}{r!} \right) \quad (2)$$

continued on next page >>>

Note: Large “n” and Small “q” is usually the case when testing for failure rates of ICs.

Now for components that display an exponential characteristic of failure such as electronic components, $q = 1 - e^{-\lambda t}$ where λ = failure rate and t = exposure time. It can also be proven that for small q , $q = 1 - e^{-\lambda t} \approx \lambda t$. (See Appendix for a proof of this fact.) Since our objective is to measure failure rate, λt is substituted for q to get:

$$P(r \text{ or less failures}) \approx \sum_{k=0}^r \frac{(n\lambda t)^k}{k!} e^{-n\lambda t} = e^{-n\lambda t} \left(1 + n\lambda t + \frac{(n\lambda t)^2}{2!} + \dots + \frac{(n\lambda t)^{r-1}}{(r-1)!} + \frac{(n\lambda t)^r}{r!} \right) \quad (3)$$

To make equation (3) easier to handle, let $u = n\lambda t$ and the result is Equation (4) as follows:

$$P(r \text{ or less failures}) \approx R(u) = e^{-u} \left(1 + u + \frac{u^2}{2!} + \dots + \frac{u^{r-1}}{(r-1)!} + \frac{u^r}{r!} \right) = e^{-u} \sum_{k=0}^r \frac{u^k}{k!} \quad (4)$$

Table Constructed based on above Redundancy Problem

Table1 is constructed listing $R(u) =$ Equation (4) and $F(u) = 1 - R(u)$ for the first five values of r of the above problem involving n redundant items. It is important to note that Equation (4) is a Reliability equation expressing the probability of success of the event “ r or less failures”, and $F(u)$ expressing the probability of failure of that event.

Table 1 will reveal a striking relationship between $F(u)$ and the famous Chi Square Table that will be discussed later on in this paper.

Table 1

Failures (r)	R(u) = P(r or less failures) Reliability Equation	F(u) = 1-R(u) (Probability of Failure Equation)
	$R(u) = e^{-u} \sum_{k=0}^r \frac{u^k}{k!} \quad (4)$	$F(u) = 1 - e^{-u} \sum_{k=0}^r \frac{u^k}{k!}$
0	e^{-u}	$1 - e^{-u}$
1	$e^{-u} [1 + u]$	$1 - e^{-u} [1 + u]$
2	$e^{-u} \left[1 + u + \frac{u^2}{2!} \right]$	$1 - e^{-u} \left[1 + u + \frac{u^2}{2!} \right]$
3	$e^{-u} \left[1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} \right]$	$1 - e^{-u} \left[1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} \right]$
4	$e^{-u} \left[1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} \right]$	$1 - e^{-u} \left[1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} \right]$

Note: $u = n\lambda t$

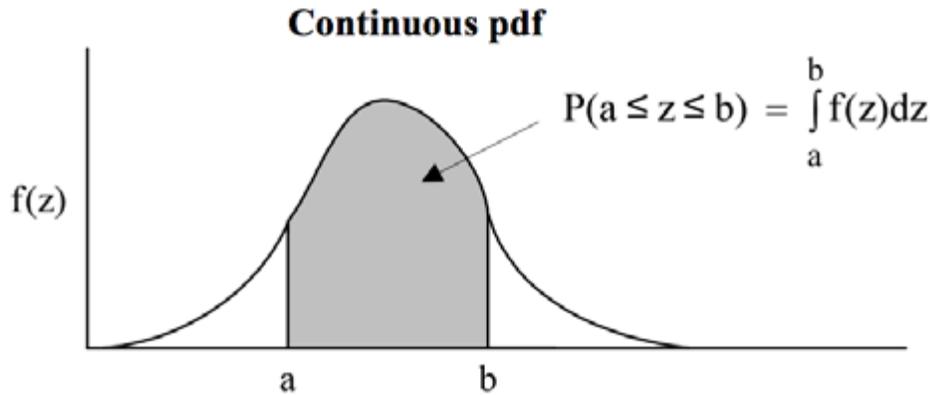
B) Definition of Probability Density Function (taken from Probability Theory)

The mathematical definition of a continuous probability density function (pdf) is a continuous function $f(z)$ that satisfies the following three properties.

- 1) The probability that z lies between two points a and b is $P(a \leq z \leq b) = \int_a^b f(x) dz$
- 2) $f(z)$ is non-negative for all real z .
- 3) The integral of the probability function $f(z)$ is one, that is $\int_{-\infty}^{+\infty} f(z) dz = 1$

Continuous pdfs are defined for an infinite number of points over a continuous interval. The probability at a single point is always zero. Probabilities are measured over intervals and not single points. Therefore, the area under the curve between two distinct points a and b defines the

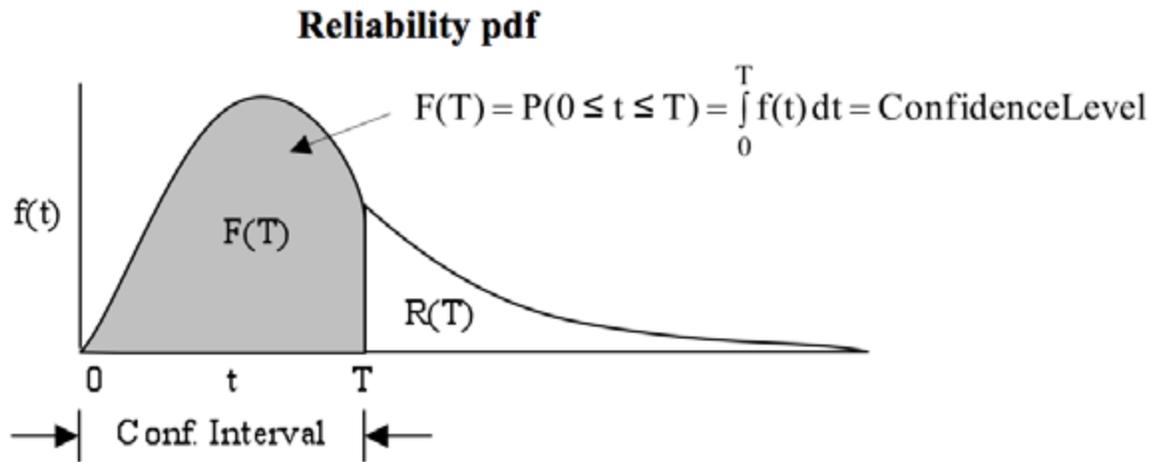
probability of that interval as shown in the shaded area below.



When applied to Reliability, $f(z)$ defines the probability of failure over an interval $a \leq z \leq b$, with the probability of a failure at a single point ($a = b$) is zero. So for example, if $z = t = \text{time}$, then $f(t)$ can answer questions like: What is probability that a component will fail between 10 and 20 hours?

C) Reliability pdf

A typical Reliability pdf would look something like the following, where $F(T)$ is the probability that a component will fail between 0 and T hours represented by the shaded area shown below. Since the entire area under a pdf is 1, the area labeled $R(T)$ must equal $1 - F(T)$ which equals the reliability or probability of success of the component at time T .



Definitions: Confidence Interval, Limit, and Level

Confidence Interval - In Reliability pdf above, $0 \leq t \leq T$ is a confidence interval.

Confidence Limit - In above confidence interval, 0 is a lower confidence limit, and T is an upper confidence limit.

Confidence Level - A percentage "measure of times" test results can be expected to be within a specified interval. In the Reliability pdf above, a percentage measure of times that the variable t will be found in interval $0 \leq t \leq T$ (probability). The Confidence Level is also mathematically defined to be the shaded area of the above Reliability pdf.

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Chi Square (χ^2) Table

The Chi Square Table is generated using what is known as Chi Square (χ^2) Equations as shown in Column 2 Table 2. Column 1 lists positive integers labeled df which stands for "Degrees of Freedom". For reasons that will soon become apparent, Column 1 only lists even integers. Column 2 lists associated χ^2 Equations defined for each df, and Column 3 contains integrals from 0 to x for each corresponding equation in Column 2. See Appendix for graphs of these equations and their integrals.

Table 2

Chi Square (χ^2) Equations		
df	χ^2 Equation f(z)	Integral of χ^2 Equation F(x)
df	$f(z) = \frac{z^{df/2-1} \cdot e^{-z/2}}{2^{df/2} \cdot (df/2 - 1)!}$	$F(x) = \int_0^x f(z) dz = 1 - e^{-x/2} \sum_{k=0}^r \frac{1}{k!} \cdot \left(\frac{x}{2}\right)^k$
2	$\frac{1}{2} e^{-z/2}$	$1 - e^{-x/2}$
4	$\frac{1}{4} z \cdot e^{-z/2}$	$1 - e^{-x/2} \left(1 + \frac{x}{2}\right)$
6	$\frac{1}{16} z^2 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2\right]$
8	$\frac{1}{96} z^3 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3\right]$
10	$\frac{1}{768} z^4 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3 + \frac{1}{4!} \left(\frac{x}{2}\right)^4\right]$

Notes:

- 1) f(z) is non - negative, and $\int_0^{\infty} f(z) dz = 1$. Therefore by definition the χ^2 equations are pdfs.
- 2) By definition $P(0 \leq z \leq x) = F(x)$
- 3) Odd integer dfs are not required for this application, and are omitted from this discussion.

Table 2 with r Added

Let $df = 2r+2$ or $r = (df-2)/2$ where r is the number of failures in the above redundancy problem. Then Table 2 takes on the form of Table 3. Now set $u = x/2$ and compare these integral equations in Column 4 with the equations of Table 1 Column 3. Conclude by inspection that these equations in Column 4 are exactly the Probability of Failure Equations of the above Redundancy problem. It is very important to note what Table 3 is revealing here. With respect to Reliability, the integrals of all χ^2 pdf equations defined for even dfs, are in fact probability of failure equations of n identical items operating parallel redundant for any number of r failures.

Table 3

Chi Square (χ^2) pdf Equations			
r	df	$f(z) = \chi^2$ pdf	$F(x) = \text{Integral of } \chi^2 \text{ pdf}$
	$2r+2$	$\frac{z^{df/2-1} \cdot e^{-z/2}}{2^{df/2} \cdot (df/2-1)!}$	$F(x) = \int_0^x f(z) dz = 1 - e^{-x/2} \sum_{k=0}^r \frac{1}{k!} \cdot \left(\frac{x}{2}\right)^k$
0	2	$\frac{1}{2} e^{-z/2}$	$1 - e^{-x/2}$
1	4	$\frac{1}{4} z \cdot e^{-z/2}$	$1 - e^{-x/2} \left(1 + \frac{x}{2}\right)$
2	6	$\frac{1}{16} z^2 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2\right]$
3	8	$\frac{1}{96} z^3 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3\right]$
4	10	$\frac{1}{768} z^4 \cdot e^{-z/2}$	$1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3 + \frac{1}{4!} \left(\frac{x}{2}\right)^4\right]$

Notes:

- Recall the area under any pdf = 1, therefore $R(x) = 1 - F(x)$ (unshaded area)
- $f(z)$ can also be expressed as $f(z) = \frac{z^r \cdot e^{-z/2}}{2^{r+1} \cdot r!}$

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D) Chi Square Table

The construction of the famous Chi Square Table (CST) itself is based on the family of χ^2 pdf equations. There are just two user inputs to the table, df and α . Mathematically stated, df selects the correct pdf and corresponding probability of failure equation, and α specifies an area. The resultant table lookup is a real number usually labeled $\chi_{1-\alpha}^2$ such that the area under the pdf from 0 to $\chi_{1-\alpha}^2$ is equal to α , i.e. the shaded area of the pdf shown in Table 4.

As an example, for a Confidence Level $\alpha = 0.95$, Table 4 lists the first 5 even df entries of the CST. The corresponding CST lookups are listed in Column 3. Close examination reveals that the table lookups are the exact solutions to the corresponding equation F(x) for each r as shown in Column 4. So for example in the case where r =1, $F(9.488) = 0.95 = 1 - e^{-4.744} (1 + 4.744)$

Table 4

Chi Square Table (CST)			
Fails	From CST	Results	
r	df	Table lookup	Probability of Failure Equation
	$(2r+2)$	$x = \chi^2_{0.05}$	$F(x) = 0.95$
0	2	5.991	$0.95 = 1 - e^{-x/2}$
1	4	9.488	$0.95 = 1 - e^{-x/2} \left(1 + \frac{x}{2}\right)$
2	6	12.592	$0.95 = 1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2\right]$
3	8	15.507	$0.95 = 1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3\right]$
4	10	18.307	$0.95 = 1 - e^{-x/2} \left[1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3 + \frac{1}{4!} \left(\frac{x}{2}\right)^4\right]$

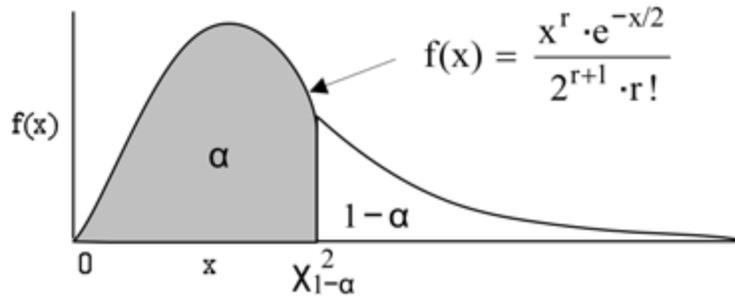
(df = Degrees of Freedom, α = Confidence Level = 0.95)

Note: See Appendix for a typical CST.

Putting it all together

Recall the basic objective was to show how $\lambda_{MAX} = \frac{\chi_{1-\alpha}^2 [with\ df = 2(r+1)]}{2T}$ is derived.

Typical Chi Square pdf



- 1) Set $df = 2r+2$ (Recall this automatically selects the correct Chi Square pdf)
- 2) Assign a Confidence Level α
- 3) From the definition of Confidence Interval of a pdf, $P(0 \leq x \leq \chi_{1-\alpha}^2) = \alpha$
- 4) Equate Table1 Col3 with Table4 Col4 and verify $u = n\lambda t = x/2 \Rightarrow x = 2n\lambda t$
- 5) Substitute $2n\lambda t$ for x and get $P(0 \leq 2n\lambda t \leq \chi_{1-\alpha}^2) = P(0 \leq \lambda \leq \frac{\chi_{1-\alpha}^2}{2nt}) = \alpha$
- 6) Set $T = nt$ (device hours). Substitute T for nt and conclude $P(0 \leq \lambda \leq \frac{\chi_{1-\alpha}^2}{2T}) = \alpha$

The derivation is complete at this point. However, it is important to note that depending on what book or article one reads, 6) can be expressed using various other notations such as:

- a) $\lambda_{MAX} = \frac{\chi_{1-\alpha}^2}{2T}$ with probability, α , or
- b) $\lambda_{MAX} = \frac{\chi_{1-\alpha}^2 [with\ df = 2(r+1)]}{2T}$ with Confidence Level α , or
- c) $\lambda_{MAX} = \frac{\chi_{1-\alpha; 2r+2}^2}{2T}$ with Confidence Level α .

Example:

50 ICs were tested for 100 hours. The test resulted in 3 failures and 47 survivors.

Calculate λ_{MAX} with a Confidence Level of 95%.

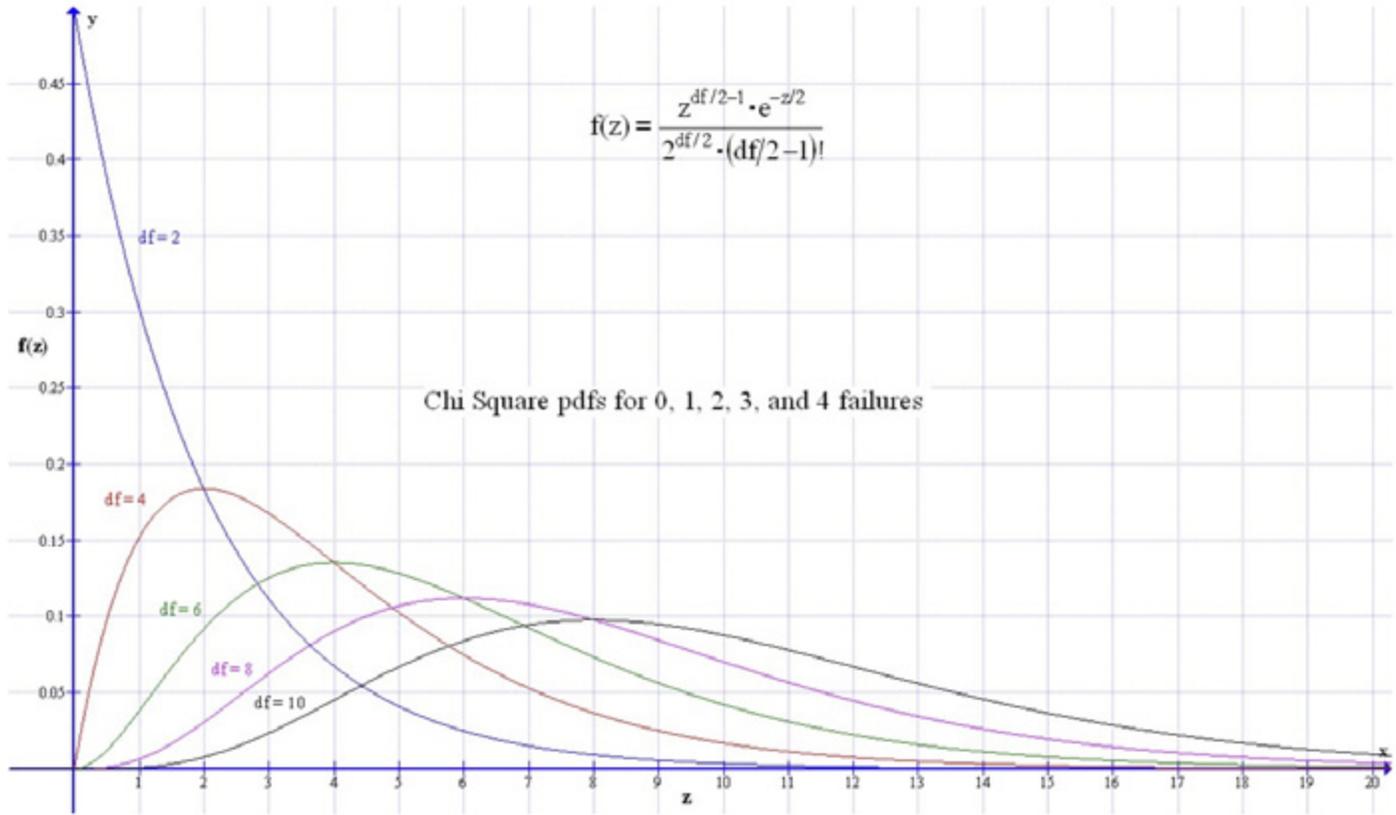
Solution: $T = nt = 50 \times 100 \Rightarrow \lambda_{MAX} = \frac{\chi_{0.05; 8}^2}{2 \times 5000} = \frac{15.507}{10,000} = 0.0015507$ failures per hour

Conclusion:

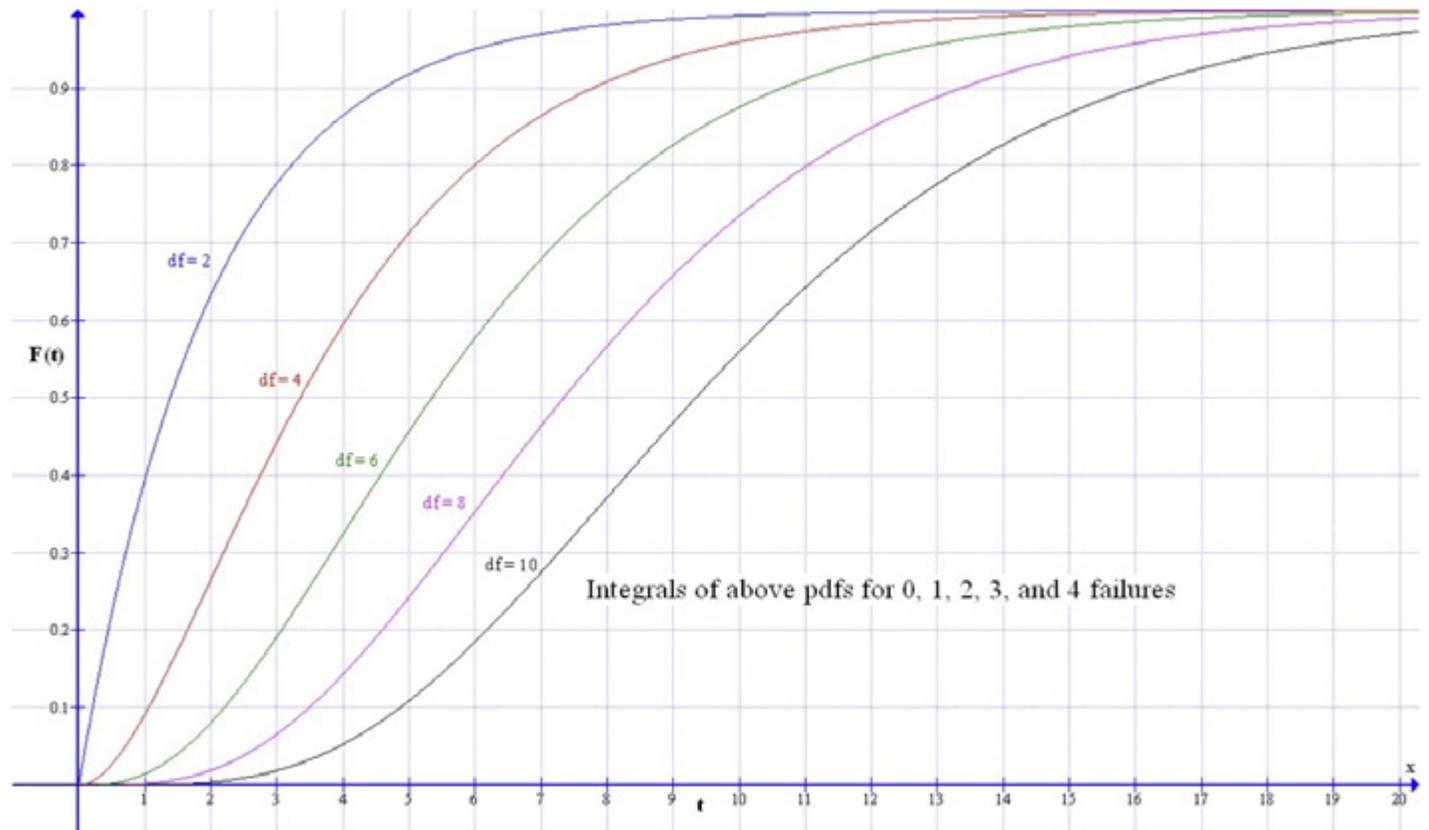
This article provided sufficient evidence to conclude that the foundation of the subject failure rate calculation algorithm is rooted in pure probability theory (mathematics). To be more precise, it is rooted in the definition of a probability density function.

Appendix

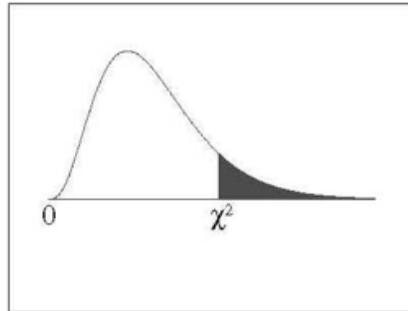
Chi-square pdf graphs for 2, 4, 6, 8, and 10 degrees of freedom



Probability of Failure Curves for 0, 1, 2, 3, and 4 failures



Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_{\alpha}$.

<i>df</i>	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

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E) Theorem: Poisson Approximation Theorem

If n is large and q is small, then $\frac{n!}{k!(n-k)!} p^{n-k} q^k \approx \frac{(nq)^k}{k!} e^{-nq}$

Proof :

$$\begin{aligned} \frac{n!}{k!(n-k)!} p^{n-k} q^k &= \frac{n(n-1)(n-2) \cdots (n-k+1)}{k!} (1-q)^{n-k} q^k \approx \frac{n^k}{k!} (1-q)^{n-k} q^k \text{ since } n \text{ is large} \\ &= \frac{n^k}{k!} \cdot \frac{(1-q)^n}{(1-q)^k} \cdot q^k \approx \frac{n^k}{k!} \cdot \frac{(1-q)^n}{1} \cdot q^k = \frac{(nq)^k (1-q)^n}{k!} \text{ since } q \text{ is small (1)} \end{aligned}$$

now compare $(1-q)^n$ with e^{-nq} by expanding both terms out.

$$\begin{aligned} (1-q)^n &= 1 - nq + \frac{n(n-1)}{2!} q^2 - \frac{n(n-1)(n-2)}{3!} q^3 + \cdots \\ &\approx 1 - nq + \frac{n^2}{2!} q^2 - \frac{n^3}{3!} q^3 + \cdots = 1 - nq + \frac{(nq)^2}{2!} - \frac{(nq)^3}{3!} + \cdots \text{ since } n \text{ is large} \\ \therefore (1-q)^n &\approx 1 - nq + \frac{(nq)^2}{2!} - \frac{(nq)^3}{3!} + \cdots \quad (2) \quad \text{and} \quad e^{-nq} = 1 - nq + \frac{(nq)^2}{2!} - \frac{(nq)^3}{3!} + \cdots \quad (3) \end{aligned}$$

comparing (2) and (3) $\Rightarrow (1-q)^n \approx e^{-nq}$ (4)

Replacing e^{-nq} for $(1-q)^n$ in (1) we get $\frac{n!}{k!(n-k)!} p^{n-k} q^k \approx \frac{(nq)^k}{k!} e^{-nq}$ //

Theorem

$$\lim_{x \rightarrow 0} 1 - e^{-x} = x$$

Proof

$$\begin{aligned} e^{-x} &= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots \Rightarrow 1 - e^{-x} = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \cdots \Rightarrow \\ 1 - e^{-x} &= x(1 - \frac{x}{2!} + \frac{x^2}{3!} - \frac{x^3}{4!} - \cdots) \text{ and } \lim_{x \rightarrow 0} (1 - \frac{x}{2!} + \frac{x^2}{3!} - \frac{x^3}{4!} - \cdots) = 1 \Rightarrow \end{aligned}$$

$$\lim_{x \rightarrow 0} 1 - e^{-x} = x(1) = x //$$

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